STUDENT'S NAME: ________________________________

TEACHER'S NAME: ________________________________

2024

HURLSTONE AGRICULTURAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK 4

TRIAL EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using a black or blue pen.
- NESA approved calculators may be used.
- A reference sheet is provided at the end of this question booklet.
- For questions in Section II, show all relevant mathematical reasoning and/or calculations.
- This examination paper is not to be removed from the examination centre.

Total marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10. The multiple-choice answer sheet has been provided at the end of this question booklet.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 7 – 14)

- Attempt Questions 11 – 16, write your solutions in the answer booklets provided. Extra working pages are available if required.
- Allow about 2 hours and 45 minutes for this section.

Disclaimer: Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2024 HSC Mathematics Extension 2 Examination.

Section 1

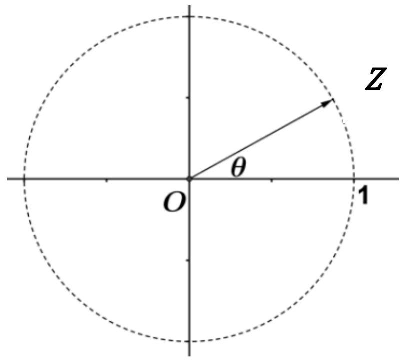
10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

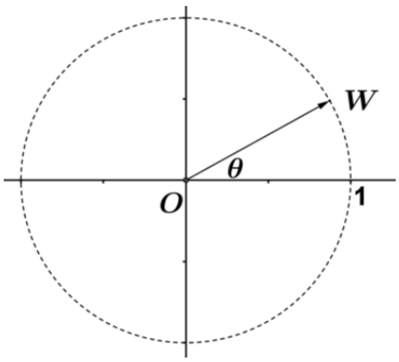
1. $Z = x + iy$ is a complex number, represented on the Argand diagram as shown below.



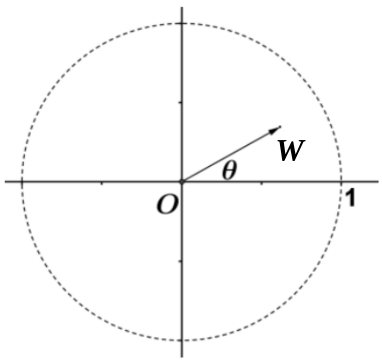
The modulus of Z is 1.

Which of the following diagrams would represent the complex number $W = \frac{1}{Z}$?

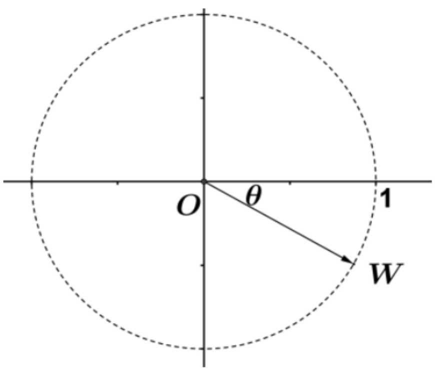
A.



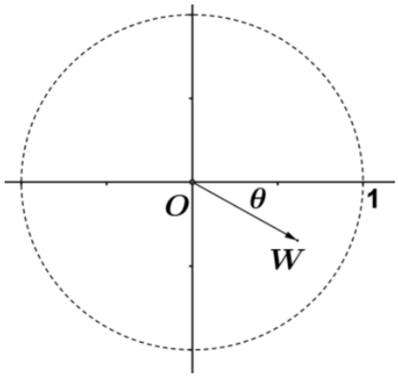
B



C.



D.



2. What value of z satisfies $z^2 = 7 - 24i$?

A. $4 - 3i$

B. $-4 - 3i$

C. $3 - 4i$

D. $-3 - 4i$

3. If $1 - i = re^{i\theta}$, what are the values of r and θ?

A. $r = \sqrt{2}\,,\ \theta = \frac{\pi}{4}$

B. $r = 2\,,\ \theta = -\frac{\pi}{4}$

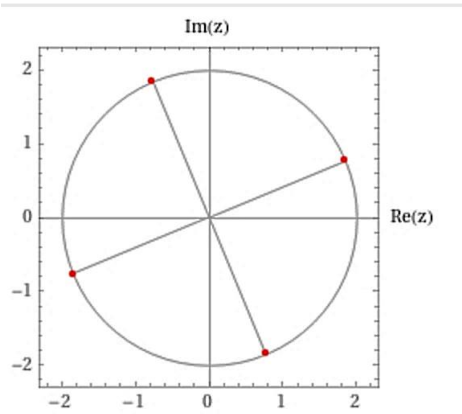
C. $r = 2\,,\ \theta = \frac{\pi}{4}$

D. $r = \sqrt{2}\,,\ \theta = -\frac{\pi}{4}$

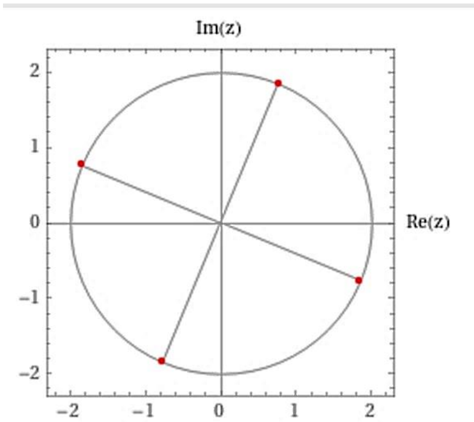
4. Which diagram shows all of the solutions to the equation $z^4 = 16i$?

(Each diagram is drawn to scale.)

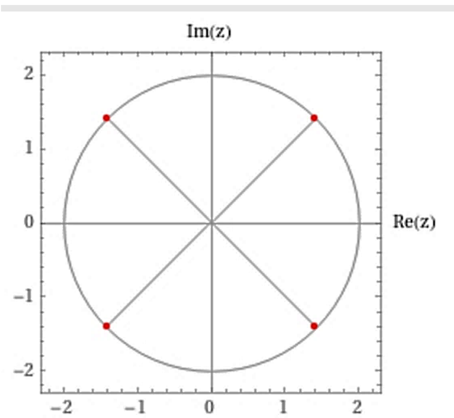
A.



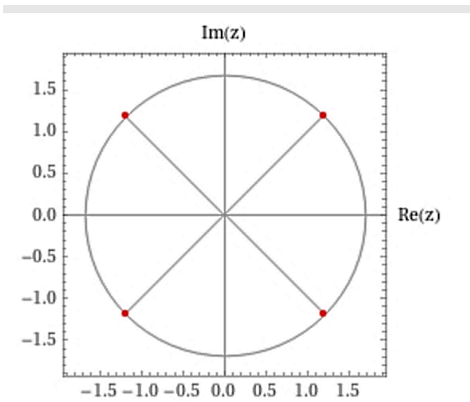
B.



C.



D.



5. Which of the following statements is true?

A. $\forall\, a, b \in \mathbb{R}$ $\quad \sin a = \sin b \Rightarrow a = b$

B. $\forall\, a, b \in \mathbb{R}$ $\quad |a + b| > |a - b|$

C. $\exists\, a, b \in \mathbb{R}$ $\quad$ such that $\ln(a + b) = \ln(ab)$

D. $\exists\, a, b \in \mathbb{R}$ $\quad |a + b| > |a| + |b|$

6. Which of the following statements about inequality proofs is true?

A. If $a > b$ and $c > d$, then $a + c > b + d$

B. If $a > b$ and $c > d$, then $a - c > b - d$

C. If $a > b$ and $c > d$, then $ac > bd$

D. If $a > b$ and $c > d$, then $\frac{a}{b} > \frac{c}{d}$

7. What is the equation of the line that satisfies the following vector equation?

$$\underset{\sim}{\boldsymbol{r}} = 3\underset{\sim}{\boldsymbol{i}} + \lambda\left(4\underset{\sim}{\boldsymbol{i}} + \underset{\sim}{\boldsymbol{j}}\right)$$

A. $y = \frac{1}{4}x + 3$

B. $y = 4x - \frac{3}{4}$

C. $y = 4x + 3$

D. $y = \frac{1}{4}x - \frac{3}{4}$

8. Which of the following is a true statement about the point $(-2, 5, -6)$ and the sphere with vector equation: $\left|\underset{\sim}{\boldsymbol{r}} - \begin{pmatrix} \mathbf{3} \\ \mathbf{4} \\ \mathbf{-2} \end{pmatrix}\right| = 7$?

A. The point is outside the sphere.

B. The point lies on the surface of the sphere

C. The point is inside the sphere, but not at its centre.

D. The point is at the centre of the sphere.

9. Without evaluating the integrals, which one of the following integrals is greater than zero?

A. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x}{2+\cos x}\, dx$

B. $\int_{-\pi}^{\pi} x^3 \sin x\, dx$

C. $\int_{-1}^{1} \left(e^{-x^2} - 1\right) dx$

D. $\int_{-2}^{2} \tan^{-1}(x^3)\, dx$

10. Which expression is equal to $\int x^2 \sin x\, dx$?

A. $-x^2 \cos x - \int 2x \cos x\, dx$

B. $-2x \cos x + \int x^2 \cos x\, dx$

C. $-x^2 \cos x + \int 2x \cos x\, dx$

D. $-2x \cos x - \int x^2 \cos x\, dx$

END OF SECTION I

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section.

Answer the questions in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

MARKS

(a) If $z = 1 + 3i$ and $w = 2 - i$, find in the form $x + iy$ where x and y are real numbers.

(i) $\bar{z}w$ **1**

(ii) $\frac{z}{w}$ **2**

(b) Given $z = \sqrt{6} - \sqrt{2}i$, find $\text{Arg}(z)$. **2**

(c) It is given that $1 + i$ is a root of $P(z) = 2z^3 - 3z^2 + rz + s$ where r and s are real numbers.

(i) Explain why $1 - i$ is a also a root of $P(z)$. **1**

(ii) Factorise $P(z)$ over the real numbers. **3**

(d) Consider the following equations:

$$|z - (3 + 2i)| = 2 \text{ and}$$

$$|z + 3| = |z - 5|.$$

(i) Draw a neat sketch of both equations on the same Argand diagram. **3**

(ii) Hence write down all the values of z which satisfy simultaneously:

$$|z - (3 + 2i)| = 2 \text{ and } |z + 3| = |z - 5|.$$ **1**

(iii) Use your diagram in (i) to determine the values of k for which the simultaneous equations

$$|z - (3 + 2i)| = 2 \text{ and } |z - 2i| = k$$

have exactly one solution for z. **2**

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

MARKS

(a) (i) Evaluate:

$$\frac{\left(2e^{\frac{-i\pi}{8}}\right)^3}{\left(e^{\frac{i\pi}{8}}\right)^7}$$

Write your answer in exponential form, using the Principal argument. **2**

(ii) Write your answer to (i) in the form $x + iy$, where x and y are real numbers. **2**

(b) Write the solutions to $z^3 = 27$ in exponential form, using Principal arguments. **2**

(c) Given that $z = e^{i\theta}$, prove that $\frac{1+z^4}{1+z^{-4}} = \cos 4\theta + i \sin 4\theta$. **2**

(d) (i) If z is a fifth root of unity, write down all of the possible values of z. **2**

(ii) Let α be the complex fifth root of unity with the smallest positive argument, and suppose that:

$$u = \alpha + \alpha^4 \text{ and } v = \alpha^2 + \alpha^3.$$

Prove that u and v satisfy the equation: $z^2 + z - 1 = 0$. **3**

(iii) Hence, find the exact value for $\cos\frac{4\pi}{5}$. **2**

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

MARKS

(a) Prove that a number is even if and only if its square is even. **2**

(b) Prove that $\sqrt[3]{2}$ is irrational. **3**

(c) Suppose that $x \geq 0$ and n is a positive integer.

(i) Show that $1 - x \leq \frac{1}{1+x} \leq 1$. **2**

(ii) Hence, or otherwise, show that:

$$1 - \frac{1}{2n} \leq n \ln\left(1 + \frac{1}{n}\right) \leq 1$$ **2**

(iii) Hence, explain why $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = e$ **1**

(d) Use Mathematical induction to prove that, for $n \geq 1$,

$$x^{(3^n)} - 1 = (x-1)(x^2+x+1)(x^6+x^3+1)\ldots\left(x^{(2\times3^{n-1})} + x^{(3^{n-1})} + 1\right)$$ **3**

(e) Let $f(x)$ be a function with a continuous derivative.

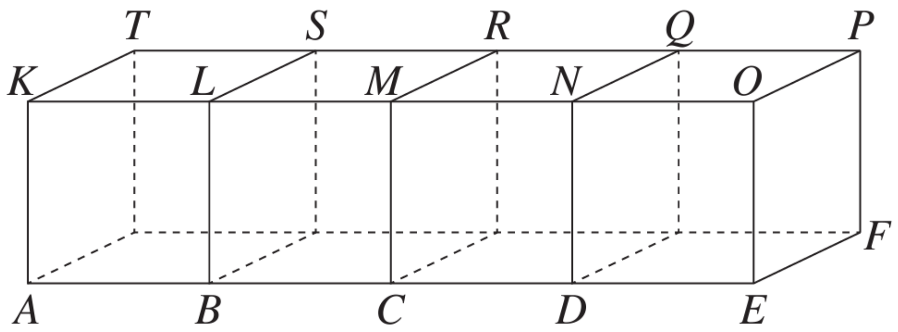
Prove that $y = \left(f(x)\right)^3$ has a stationary point at $x = a$ if $f(a) = 0$ or $f'(a) = 0$. **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

MARKS

(a) Four identical cubes are placed in a line as shown in the diagram.



Give single vectors as answers to the following.

(i) $\overrightarrow{AS} + 2\,\overrightarrow{SR}$ **1**

(ii) A vector equivalent to: $\overrightarrow{AB} + \overrightarrow{DP}$ **1**

(b) A line l has vector equation $\underset{\sim}{r} = 7\underset{\sim}{i} + 2\underset{\sim}{j} - 9\underset{\sim}{k} + \lambda\left(3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}\right)$.

Find the point of intersection of line l and the xz plane. **2**

(c) A sphere has centre $(2, -3, 4)$ and radius 5 units.

(i) Write down a vector equation for the sphere. **1**

(ii) Write down a Cartesian equation for the sphere. **1**

(iii) Find the points of intersection of the sphere and the line:

$$\underset{\sim}{r} = \begin{pmatrix} -4 \\ -3 \\ 12 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$$ **3**

Question 14 continues on the next page.

MARKS

(d) Let $ABCD$ form the vertices of a rectangle. Let $\overrightarrow{AB} = \underset{\sim}{\boldsymbol{a}}$ and $\overrightarrow{BC} = \underset{\sim}{\boldsymbol{b}}$

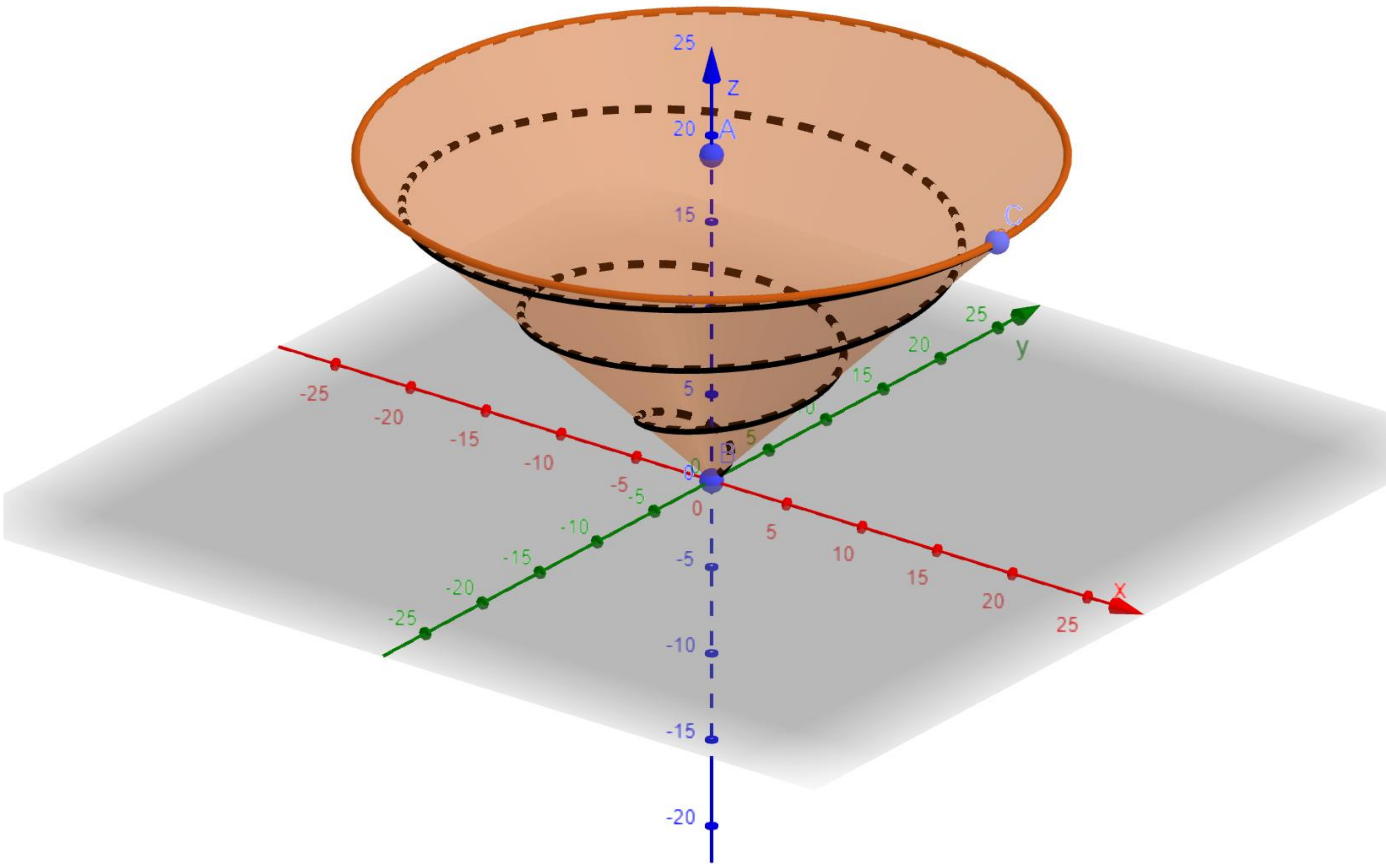
Let P, Q, R and S be the midpoints of AB, BC, CD and AD respectively.

Use vector methods to prove that $PQRS$ is a rhombus. **3**

(e) A curve Φ spirals 3 times around the inverted cone as shown.

The cone has its apex at the origin. The point $(0, 0, 6\pi)$ is at the centre of the cone's circular base, and the cone's maximum radius is also 6π units.

A particle is initially at the origin and moves along the curve Φ on the surface of the cone, ending at the point $(6\pi, 0, 6\pi)$.



Give a possible set of parametric equations that describe the curve Φ. **3**

End of Question 14

Question 15 (15 marks) Use the Question 15 Writing Booklet.

MARKS

(a) (i) Find real numbers a and b such that

$$\frac{x^2-7x+4}{(x+1)(x-1)^2} \equiv \frac{a}{x+1} + \frac{b}{x-1} + \frac{c}{(x-1)^2}\,.$$ **2**

(ii) Hence, find $\int \frac{x^2-7x+4}{(x+1)(x-1)^2}\,dx$. **2**

(b) Evaluate $\int_0^{\frac{1}{2}} (3x-1)\cos(\pi x)\,dx$. **3**

(c) (i) Using a suitable substitution, show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. **1**

(ii) A function $f(x)$ has the property that $f(x) + f(a-x) = f(a)$.

Using part (i), or otherwise, show that

$$\int_0^a f(x)dx = \frac{a}{2} f(a).$$ **2**

(d) Let $I_n = \int_0^1 \frac{x^{2n}}{x^2+1}\,dx$, where n is an integer and $n \geq 0$.

(i) Show that $I_0 = \frac{\pi}{4}$. **1**

(ii) Show that $I_n + I_{n-1} = \frac{1}{2n-1}$. **2**

(iii) Hence, or otherwise, find $\int_0^1 \frac{x^4}{x^2+1}\,dx$. **2**

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet.

MARKS

(a) A particle starts at the origin with velocity 1 and acceleration given by

$$a = v^2 + v,$$

where v is the velocity of the particle.

Find an expression for x, the displacement of the particle, in terms of v. **3**

(b) A particle moves along a straight line with displacement x m and velocity v ms^{-1}.
The acceleration of the particle is given by

$$\ddot{x} = 2 - e^{-\frac{x}{2}}.$$

Given that $v = 4$ when $x = 0$, express v^2 in terms of x. **3**

(c) An object is moving in simple harmonic motion along the x-axis.
The acceleration of the object is given by $\ddot{x} = -4(x - 3)$ where x is its displacement from the origin, measured in metres, after t seconds.

Initially, the object is 5.5 metres to the right of the origin and moving towards the origin. The object has a speed of 8 ms^{-1} as it passes through the origin.

(i) Between which two values of x is the particle oscillating? **2**

(ii) Find the first value of t for which $x = 0$, giving the answer correct to 2 decimal places. **2**

Question 16 continues on the next page.

MARKS

(d) A particle is moving along the x-axis in simple harmonic motion. The position of the particle is given by

$$x = \sqrt{2}\cos 3t + \sqrt{6}\sin 3t, \text{ for } t \geq 0.$$

(i) Write x in the form $R\cos(3t - \alpha)$, where $R \geq 0$ and $0 < \alpha < \frac{\pi}{2}$. **2**

(ii) Find the two values for x where the particle comes to rest. **1**

(iii) When is the first time that the speed of the particle is equal to half of its maximum speed? **2**

End of Question 16

End of examination

2024 Yr12 HSC Assessment Task 4	
Multiple Choice Solutions and Marking Guidelines	
Solutions	**Marking Guidelines**

1) Since the modulus is 1, when realising the denominatior of $\frac{1}{z}$ we get the conjugate.

$$\frac{1}{z} = \frac{1}{x+yi} = \frac{x-yi}{x^2+y^2} = \frac{x-yi}{1} = x - yi$$

Hence, the answer is option C.

2) Option A, since:

$$(4-3i)^2 = 16 - 2 \times 12i - 9 = 7 - 24i$$

Question 3:

Modulus $= -\frac{\pi}{4}$; argument $= \sqrt{2}$ Answer: D

Question 4

The 4th root of a number with moduus 16 will have modulus 2. Hence option D is eliminated.

The argument of $16i$ is $\frac{\pi}{4}$ so one 4th root will have arg $= \frac{\pi}{16}$.

The other 3 roots are equally spaced around the Argand Diagram. Answer A

5) Option A is false since sine has equal values for different angles.

Option B is false when $a > 0$ and $b < 0$.

Option D is false because of the triangle inquality.

Hence it must be option C, when $a = b = 0$.

6) Option A is true.

If $a > b$ and we add c, we have $a + c > b + c$ (property of inequalities).
Then, if $c > d$ and we replace c on the RHS, the inequality is still true.

Question 7

The direction vector gives a gradient of $\frac{1}{4}$. When $\lambda = 0$ the line is at $(3,0)$. Hence the y-intercept is at $\left(0, -\frac{3}{4}\right)$. Answer D

Question 8

The distance from the centre of the sphere to the point is $\sqrt{5^2 + 1^2 + 4^2} = \sqrt{42} < 7$.

Answer C

Year 12	Mathematics Extension 2	Ass Task 3 2024 HSC
Multiple choice	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MEX 12-5 Applies techniques of integration to structured and unstructured problems.

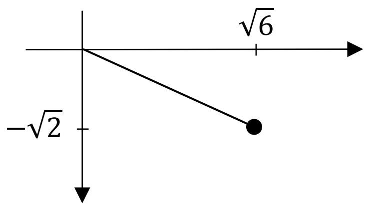
Question/ Outcome	Solutions	Marking Guidelines
9 **12-5**	Option A: $f(-x)=\frac{-x}{2+\cos(-x)}=-\frac{x}{2+\cos x}=f(x)$ ie odd function, so integral is 0. Option D: same reasoning as A Option C: the graph is entirely below the x-axis, so negative integral. Option B: $x^3\sin x$ is non-negative in the given integral, and even function, hence the integral is positive.	1 mark: B
10 **12-5**	$u=x^2$ $\quad$ $dv=\sin x\,dx$ $du=2xdx$ $\quad$ $v=-\cos x$ $uv-\int v\,du=-x^2\cos x+2\int x\cos x\,dx$	1 mark: D

2024 Yr12 HSC Assessment Task 4

Question 11 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MEX12-4
uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems

Solutions	**Marking Guidelines**
a) i) $\bar{z}w = (1-3i)(2-i)$ $= (2 - i - 6i - 3$ $= -1 - 7i$	1 Mark Correct solution
ii) $\frac{z}{w} = \frac{1+3i}{2-i} \times \frac{2-i}{2-i}$ $= \frac{-1+7i}{5}$ $-\frac{1}{5} + \frac{7i}{5}$	2 Marks Correct solution 1 Mark Attempts to realise the denominator
b) $\tan\alpha = \frac{\sqrt{2}}{\sqrt{6}} = \frac{1}{\sqrt{3}}$ $\alpha = \frac{\pi}{6}$  $\arg(z) = -\frac{\pi}{6}$	2 Marks Correct solution 1 Mark Correctly finds magnitude of argument

c) i) If $z_1$is a root of the polynomial, then $\overline{z_1}$ is also a root (polynomial has real coefficients).

$$\overline{z_1} = 1 - i$$

1 Mark
Correct explanation that mentions real coefficients

ii)

$$P(z) = 2z^3 - 3z^2 + rz + s$$

$$= \big(z - (1+i)\big)\big(z - (1-i)\big)(az + b)$$

$$= (z^2 - 2z + 2)(az + b)$$

$$= az^3 - (2a - b)z^2 + (2a - 2b)z + 2b$$

$$\equiv 2z^2 - 3z^2rz + s$$

Hence, we have:

$$a = 2.$$

$$2a - b = 3$$
$$b = 1$$

$$r = 2a - 2b = 2$$

$$s = 2b = 2$$

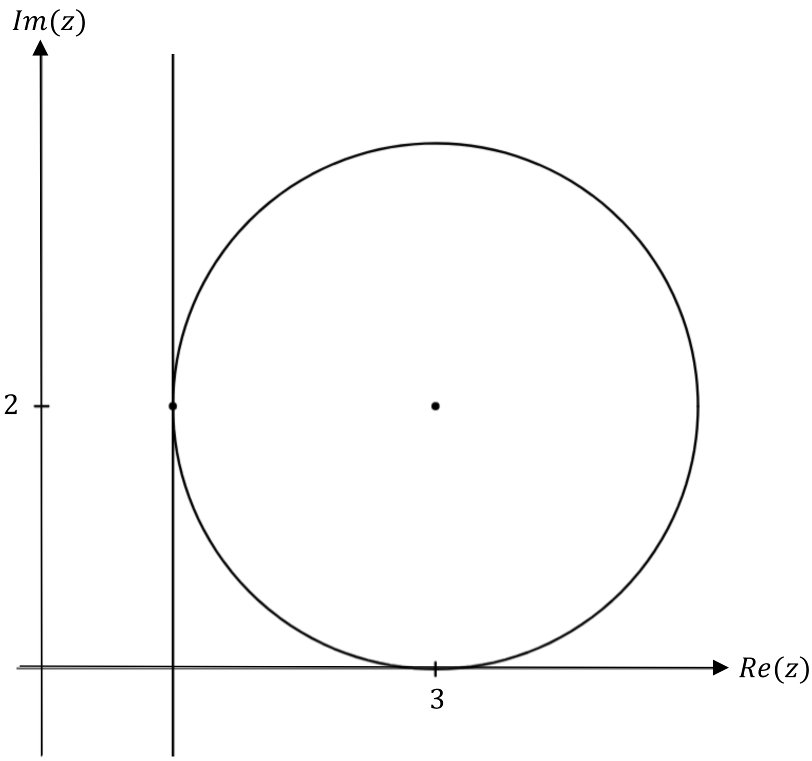
Hence, $P(z) = (z^2 - 2z + 2)(2z + 1)$ over the reals.

3 Marks
Correct solution

2 Marks
Significant progress to establishing correct factorisation

1 Marks
Factorisation with complex roots

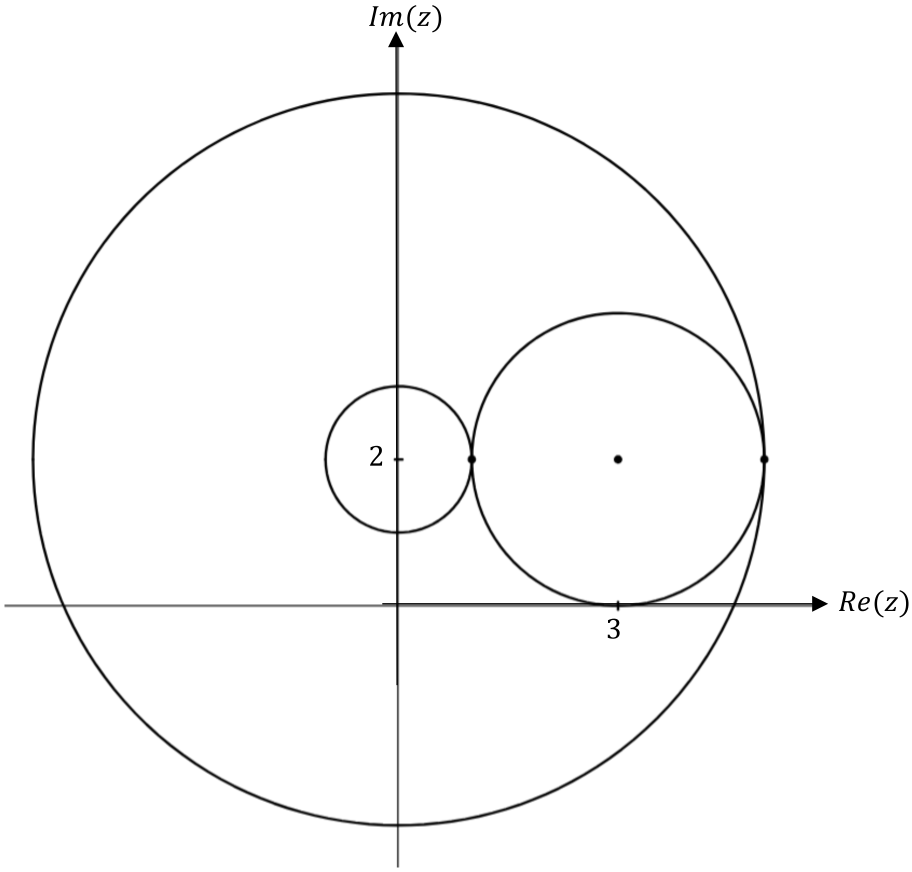
d) i)



3 Marks
Correct solution including circle, line and intersection point

2 Marks
Two of the above three

1 Marks
Some progress to establishing correct factorisation

ii) The line is a tangent to the circle. Hence, the only point of intersection is $z = 1 + 2i$.	1 Mark Correct solution
iii) $\lvert z - 2i \rvert = k$ represents a circle with centre $2i$ and radius k. Hence, $\lvert z - 2i \rvert = k$ and $\lvert z - (3 + 2i) \rvert = 2$ have one solution when $k = 1$ and 5.  **Note:** one solution for z means one intersection point. There are two values of k for which this happens.	2 Marks Correct solution 1 Mark Find only one value for k

Question 12

Outcomes Addressed in this Question

MEX12-4 uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems

Question	Solutions	Marking Guidelines
(a)(i)	(i) $\frac{\left(2e^{\frac{-i\pi}{8}}\right)^3}{\left(e^{\frac{i\pi}{8}}\right)^7} = 8e^{\frac{-5\pi i}{4}} = 8e^{\frac{3\pi i}{4}}$ (Principal Arg)	**(a)(i) 2 marks**: Correct solution with Principal Arg. **1 mark:** Considerable relevant progress.
(ii)	(ii) $8\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = 8\left(\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$ $= -4\sqrt{2} + 4\sqrt{2}\,i$	**(ii) 2 marks**: Correct solution. **1 mark:** Considerable relevant progress.
(b)	$z = 3, 3e^{\frac{2\pi 1}{3}}, 3e^{-\frac{2\pi 1}{3}}$ Alternate form: $z = 3, 3cis\frac{2\pi}{3}, 3cis\frac{-2\pi}{3}$	**(b) 2 marks**: All 3 roots correct, including Principal Args for the complex roots. **1 mark**: At least 1 root correct as above.
(c)	$\frac{1+z^4}{1+z^{-4}} = \frac{z^4(z^{-4}+1)}{1+z^{-4}} = z^4$ Using DeMoivre's theorem for values with modulus 1, $z^4 = \cos 4\theta\ \ i\sin 4\theta$	**(c) 2 marks:** Correct solution with working. **1 mark:** At least one correct manipulation of the fraction.
(d) (i)	(i) $z = e^{\frac{2k\pi i}{5}}, k = 0, \pm 1, \pm 2$ Alternate form: $z = 1, cis \pm\frac{2\pi}{5}, cis\ \pm\frac{4\pi}{5}$	**(d) (i) 2 marks:** Correct responses in any form. **1 mark:** At least 2 roots correct.
(ii)	(ii) $\arg(\alpha) = \frac{2\pi}{5}$ and roots of $z^5 - 1 = 0$ are: $1, \alpha, \alpha^2, \alpha^3, \alpha^4$ which hence have a sum equal to $\frac{-b}{a} = 0$. To prove that u and v satisfy the quadratic, consider their sum and product. Sum $= \alpha + \alpha^4 + \alpha^2 + \alpha^3 = -1$ since the sum of all 5 roots above is 0. Product $= (\alpha + \alpha^4)(\alpha^2 + \alpha^3) = \alpha^3 + \alpha^4 + \alpha^6 + \alpha^7$ $= \alpha^3 + \alpha^4 + \alpha + \alpha^2$ since $\alpha^5 = 1$ $= -1$ as shown above. Therefore $u + v = -1$ and $uv = -1$, so u and v are roots of $z^2 + z - 1 = 0$ Note: 1. The method of sum and product uses much easier calculations than substituting into the quadratic. 2. The question gave the opportunity to just deal with α, but many chose the difficult route of changing back to powers of e or cos + $i$sin form.	**(ii) 3 marks:** Correct response including reasoning **2 marks:** Almost complete response. **1 mark:** Significant relevant progress.

(iii)	$$v = \alpha^2 + \alpha^3$$ These are conjugate pairs, so their sum is 2 Real $\left(cis\ \frac{4\pi}{5}\right)$=2 $\cos\frac{4\pi}{5}$ which is the negative root of the quadratic. $$\therefore 2\cos\frac{4\pi}{5} = \frac{-1-\sqrt{5}}{2}$$ $$\cos\frac{4\pi}{5} = \frac{-1-\sqrt{5}}{4}$$	**(iii) 2 marks**: Correct solution. **1 mark:** Correct addition of conjugate pair or equivalent. **0 marks:** Solving the quadratic (Yr9 level) without connecting it to the other parts of the question.

2024 Yr12 HSC Assessment Task 4
Question 13 Solutions and Marking Guidelines
Outcomes Addressed in this Question
MEX12-2 Chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings.

Solutions	**Marking Guidelines**
a) First prove that if a number is even then it's square is even. Let p be even, so that $p = 2k$, where k is an integer. Then, $$\begin{aligned} p^2 &= (2k)^2 \\ &= 4k^2 \\ &= 2(2k^2) \end{aligned}$$ Which is even. Hence if p is even then p^2 is even. Now prove that if the square of a number is even then the number is even (i.e. the converse). Let q^2 be even, such that $q^2 = 2k$, where k is an integer. Then, $$\frac{q \times q}{2} = k$$ So 2 divides either q or q, that is 2 divides q as k is an integer. Hence, if q^2 is even then q is even. Therefore, a number is even if and only if the number squared is even.	2 Marks Correct solution 1 Mark Partially correct proof that does not address "if and only if" implication
b) Assume that $\sqrt[3]{2}$ is rational. $\sqrt[2]{3} = \frac{p}{q}$ where p and q are co-prime $$2 = \frac{p^3}{q^3}$$ $$2q^3 = p^3$$ Hence, p^3 is even. This implies p is even. This means that $p^3 = 2k$, where k is an integer.	3 Marks Correct solution 2 Marks Mostly correct proof 1 Mark Establishes some aspects of a proof by contradiction

So $$2q^3 = (2k)^3$$ $$2q^3 = 8k^3$$ $$2q^3 = 8k^3$$ $$q^3 = 2 \times 2k^3$$ $$q^3 = 2n \qquad \text{where } n \text{ is an integer}$$ Hence, q^3 is even. This implies q is even. Thus, p and q are not co-prime and the initial assumption is contradicted. Hence, $\sqrt[3]{2}$ is irrational.	
c) i) Since for $x \geq 0$ we have $$1 - x^2 \leq 1$$ $$(1-x)(1+x) \leq 1$$ Since $(1+x) > 0$ we have $$1 - x \leq \frac{1}{1+x}$$ And since $x \geq 0$ then $1 + x \geq 1$ Hence $$\frac{1}{1+x} \leq 1$$ And $$1 - x \leq \frac{1}{1+x} \leq 1$$	2 Marks Correct solution 1 Mark Attempts to establish inequality from a valid known inequality
ii) Since $$\int_a^b \frac{1}{1+x} dx = [\ln(1+x)]_a^b = \ln(1+b) - \ln(1+a)$$ We want $\ln\left(1+\frac{1}{n}\right)$, so let $a = 0$ and $b = \frac{1}{n}$ where $n > 0$. Using part (i) and integrating, we have $$\int_0^{\frac{1}{n}} 1 - x dx \leq \int_0^{\frac{1}{n}} \frac{1}{1+x} dx \leq \int_0^{\frac{1}{n}} 1 dx$$ $$\left[x - \frac{x^2}{2}\right]_0^{\frac{1}{n}} \leq [\ln(1+x)]_0^{\frac{1}{n}} \leq [x]_0^{\frac{1}{n}}$$	2 Marks Correct Solution 1 Mark Establishes an integral that could lead to correct solution

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$$\frac{1}{n} - \frac{1}{2n^2} \le \ln\left(1 + \frac{1}{n}\right) \le \frac{1}{n}$$

Multiplying both sides by n.

$$1 - \frac{1}{2n} \le n\ln\left(1 + \frac{1}{n}\right) \le 1$$

iii) From (ii)

$$1 - \frac{1}{2n} \le n\ln\left(1 + \frac{1}{n}\right) \le 1$$

$$1 - \frac{1}{2n} \le \ln\left(1 + \frac{1}{n}\right)^n \le 1$$

$$\lim_{n\to\infty} 1 - \frac{1}{2n} \le \lim_{n\to\infty} \ln\left(1 + \frac{1}{n}\right)^n \le \lim_{n\to\infty} 1$$

$$1 \le \lim_{n\to\infty} \ln\left(1 + \frac{1}{n}\right)^n \le 1$$

Hence

$$\lim_{n\to\infty} \ln\left(1 + \frac{1}{n}\right)^n = 1$$

So

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$$

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d) Test for base case $n = 1$

$$LHS = x^3 - 1$$

$$\begin{aligned} RHS &= (x-1)(x^2 + x + 1) \\ &= x^3 - 1 \\ &= LHS \end{aligned}$$

Assume true from $n = k$, that is

$$x^{3^k} - 1 = (x-1)(x^2 + x + 1) \ldots \left(x^{2\times 3^{k-1}} + x^{3^{k-1}} + 1\right)$$

Prove true for $n = k + 1$

$$x^{3^{k+1}} - 1 = (x-1)(x^2 + x + 1) \ldots \left(x^{2\times 3^{k}} + x^{3^{k}} + 1\right)$$

$$RHS = (x-1)(x^2 + x + 1) \ldots \left(x^{2\times 3^{k-1}} + x^{3^{k-1}} + 1\right)\left(x^{2\times 3^{k}} + x^{3^{k}} + 1\right)$$

$$= \left(x^{3^k} - 1\right)\left(x^{2\times 3^{k}} + x^{3^{k}} + 1\right)$$

$$= \left(x^{3^k}\right)^3 - 1$$

$$= x^{3^{k+1}} - 1$$

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3 Marks
Correct solution

2 Marks
Correct base case with inductive step attempted

OR
Correct inductive step with incorrect base case

1 Mark
Shows some parts of a proof by induction that could lead to a correct solution

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e) $$y = \left(f(x)\right)^3$$ $$y' = 3f(x)^2 \times f'(x)$$ There will be a stationary point if $y' = 0$, that is if $$3f(x)^2 \times f'(x) = 0$$ So $f(x) = 0$ or $f'(x) = 0$ Hence if $f(a) = 0$ or $f'(a) = 0$ then there is a stationary point at $x = a$.	2 Marks Correct solution 1 Mark Finds correct derivative

Solutions and Marking Guidelines

Question 14

Outcomes Addressed in this Question

MEX12-3 uses vectors to model and solve problems in two and three dimensions.

Part	Solutions	Marking Guidelines
(a)(i)	(i) $\overrightarrow{AQ}$	**(a)(i) 1 mark:** Correct answer
(ii)	(ii) Any of: $\overrightarrow{AR}$, $\overrightarrow{BQ}$, $\overrightarrow{CP}$	**(ii) 1 mark:** Correct answer.
(b)	*"xz* plane" means that y=0. Therefore $2+\lambda=0 \rightarrow \lambda=-2$ Intersection is at: $$\begin{pmatrix} 7-2(3) \\ 2-2(1) \\ -9-2(-2) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$$	**(b) 2 marks:** Correct answer from correct λ. **1 mark:** 1 component performed correctly.
(c) (i)	$$(i)\ \left\| \underset{\sim}{r} - \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \right\| = 5$$	**(c) (i) 1 mark:** Correct answer.
(ii)	(ii) $(x-2)^2+(x+3)^2+(z-4)^2=25$	**(ii) 1 mark:** Correct answer.
(iii)	(iii) Substitute line into sphere equation: $$(-4+3\lambda-2)^2+(-3+3)^2+(12-4\lambda-4)^2=25$$ $$(3\lambda-6)^2+(8-4\lambda)^2=25$$ $$25(\lambda-3)(\lambda-1)=0$$ $$\lambda=1,3$$ Intersection points: $$\begin{pmatrix} -4+3 \\ -3+0 \\ 12-4 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ 8 \end{pmatrix} \text{ and } \begin{pmatrix} -4+3(3) \\ -3+3(0) \\ 12-3(4) \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix}$$	**(iii) 3 marks:** Merge the two equations and simplify; calculate λ; find intersection points. **2 marks:** 2 components performed correctly. **2 marks:** 1 correct point from only one correct λ. **1 mark:** Significant relevant progress.
(d)	$$\overrightarrow{PQ} = \frac{1}{2}\left(\underset{\sim}{a}+\underset{\sim}{b}\right) = \overrightarrow{SR}$$ Therefore, *PQRS* is a parallelogram because it has two opposite sides that are equal and parallel. $$\overrightarrow{PR} = \frac{1}{2}\underset{\sim}{a}+\underset{\sim}{b}-\frac{1}{2}\underset{\sim}{a}=\underset{\sim}{b}$$ $$\overrightarrow{SQ} = \frac{1}{2}\underset{\sim}{b}+\underset{\sim}{a}-\frac{1}{2}\underset{\sim}{b}=\underset{\sim}{a}$$ So: $\overrightarrow{PR}\cdot\overrightarrow{SQ}=\underset{\sim}{a}\cdot\underset{\sim}{b}=0$ because adjacent sides of a rectangle are perpendicular. Hence $PR \perp SQ$. *PQRS* is a rhombus because it is a parallelogram with perpendicular diagonals. Note: 1. You only need to prove one pair of equal vectors for a parallelogram. Lots of unnecessary extra work was done here.	**(d) 3 marks:** Proof of parallelogram; use of dot product to determine extra property; conclusion based upon the working shown. **2 marks:** One of the above components incomplete. **1 mark:** Proof of parallelogram only.

	2. The direction of a vector is important. $\underset{\sim}{a} - \underset{\sim}{b} \neq \underset{\sim}{a} + \underset{\sim}{b}$ without reason. Many responses did not acknowledge this. 3. Bisecting diagonals is a property of a parallelogram, so you did not need to prove bisection after showing $PQRS$ is a parallelogram.	
(e)	$$\begin{pmatrix} t\cos t \\ t\sin t \\ t \end{pmatrix}, 0 \leq t \leq 6\pi$$ Notes: 1. Cos and sin needed to be in this format, or the spiral would go in the wrong direction. 2.Some good attempts did not meet the criteria of a conical shape. E.g. if there was a constant coefficient for x and y it makes a cylinder; Other coefficients of t in the trig coordinates meant that there was the incorrect number of revolutions.	**(e) 3 marks:** Correct components and limits on the value of t. **2 marks:** Correct components. **1 mark:** At least 1 correct component. **1 mark:** Use of cos and sin for x and y components.

Year 12	Mathematics Extension 2	Ass Task 4 2024 HSC
Question No. 15	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MEX 12-5 Applies techniques of integration to structured and unstructured problems.

A note on questions 15 & 16

All parts were taken from past HSC papers, with the intended purpose of "quality control", and to aid with your preparation for what is looked for by HSC markers.
The HSC examiners notes from those questions have been included here, so that you can see what they highlighted when marking, and compare with your own responses.
In particular, if you did not attain full marks for any part, you should read the examiners comments, as it was noted in the marking of this 2024 cohort that there were a lot of commonalities with errors or misjudgments between the past and the present.

Part / Outcome	Solutions	Marking Guidelines
(a)(i)	$$x^2-7x+4\equiv a(x-1)^2+b(x+1)(x-1)+c(x+1)$$ $$\begin{array}{llll} x=1 & \to & 1-7+4=2c & \Rightarrow\ c=-1 \\ x=-1 & \to & 1+7+4=4a & \Rightarrow\ a=3 \\ x=0 & \to & 4=3-b-1 & \Rightarrow\ b=-2 \end{array}$$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution *Note: this question was marked harshly for the second mark. ie small (seemingly insignificant) calculation errors generally prevented access to full marks*
(a)(ii)	$$\int\frac{x^2-7x+4}{(x+1)(x-1)^2}\,dx=\int\left(\frac{3}{x+1}-\frac{2}{x-1}-\frac{1}{(x-1)^2}\right)dx$$ $$=3\ln\lvert x+1\rvert-2\ln\lvert x-1\rvert+\frac{1}{x-1}+c$$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution

<u>2004 HSC examiners comments</u>

(i) This part was generally well done. Most successful responses either equated coefficients or substituted and $x=-1$ after having determined the identity $x^2-7x+4\equiv a(x-1)^2+b(x+1)(x-1)-(x+1)$. A number of candidates made minor errors in equating coefficients or in substituting into the identity.

(ii) Most candidates were able to correctly find $\int\left(\frac{a}{x+1}+\frac{b}{x-1}-\frac{1}{(x-1)^2}\right)dx$ using their values of a and b. Most incorrect responses occurred when candidates were unable to integrate $(x-1)^{-2}$.

Part / Outcome	Solutions	Marking Guidelines
(b)	$$\int_0^{\frac{1}{2}}(3x-1)\cos(\pi x)\,dx \qquad \left\lvert\ \begin{array}{ll} u=3x-1 & dv=\cos(\pi x)\,dx \\ du=3dx & v=\frac{1}{\pi}\sin(\pi x)\end{array}\right.$$ $$=\left[(3x-1)\frac{1}{\pi}\sin\pi x\right]_0^{\frac{1}{2}}-\frac{3}{\pi}\int_0^{\frac{1}{2}}\sin(\pi x)\,dx$$ $$=\frac{1}{\pi}\left[\frac{1}{2}\sin\frac{\pi}{2}+\sin 0\right]-\frac{3}{\pi}\left[-\frac{1}{\pi}\cos\pi x\right]_0^{\frac{1}{2}}$$ $$=\frac{1}{\pi}\left[\frac{1}{2}-0\right]+\frac{3}{\pi^2}\left[\cos\frac{\pi}{2}-\cos 0\right]$$ $$=\frac{1}{2\pi}+\frac{3}{\pi^2}[0-1]\ =\ \frac{1}{2\pi}-\frac{3}{\pi^2}$$	<u>3 marks</u>: correct solution <u>2 marks</u>: substantially correct solution *Correctly uses limits in one of two terms, or equivalent merit* <u>1 marks</u>: partially correct solution *Attempts integration by parts, or equivalent merit*

2014 HSC examiners comments

Common problems were:

- splitting the integral into two separate integrals then applying integration by parts to one
- not selecting u and v' correctly
- incorrectly calculating v when $v' = \cos \pi x$ (typical incorrect responses included $v = \pi \sin \pi x,\ -\frac{1}{\pi}\sin \pi x$ or $\pi \sin x$)

(c)(i)	$\int_0^a f(x)dx$ $= \int_a^0 f(a-u)(-du) \quad (\times(-1) = \text{flip } limits)$ $= \int_0^a f(a-u)du$ $= \int_0^a f(a-x)dx$ $u = a - x$ $du = -dx$ $x = 0 \Rightarrow u = a$ $x = a \Rightarrow u = 0$	1 mark: correct solution
(c)(ii)	$\int_0^a f(x)dx = \frac{1}{2}\left[\int_0^a f(x)dx + \int_0^a f(a-x)dx\right]$ $= \frac{1}{2}\int_0^a \big(f(x) + f(a-x)\big)dx$ $= \frac{1}{2}\int_0^a f(a)dx$ $= \frac{1}{2}f(a)\int_0^a dx \quad NB: f(a) = constant$ $= \frac{1}{2}f(a)[x]_0^a = \frac{1}{2}f(a)[a-0]$ $= \frac{a}{2}f(a)$	2 marks: correct solution 1 mark: substantially correct solution

2007 HSC examiners comments

(a) (i) Many candidates who did not attempt this part were able to do part (ii). Candidates are advised to indicate clearly which side of an equation they are working on and to show clearly that the LHS becomes the RHS or vice versa – all steps must be shown. Also, candidates are reminded that it is not a proof to let $f(x)$ be a particular function, for example using x or x^2 to establish a result.

(ii) A variety of successful approaches were demonstrated. However, a common error when integrating the identity satisfied by f was the omission of the integration of $f(a)$, ie

$f(x) + f(a-x) = f(a) \quad \therefore \int_0^a f(x)\,dx + \int_0^a f(a-x)\,dx = "f(a)"$. Another error was to state:

as $f(x) = f(a-x)$ hence $f(x) = \frac{1}{2}f(a)$ then $\int_0^a f(x)\,dx = \frac{1}{2}af(a)$ or $\frac{1}{2}f(a)$.

Candidates are advised not to omit the dx or the du in their working, as it was frequently absent in their setting out and it is crucial for change of variables concept.

	Question 15 continued…	
(d)(i)	$I_n = \int_0^1 \frac{x^{2n}}{x^2+1}\,dx$ $I_0 = \int_0^1 \frac{x^0}{x^2+1}\,dx$ $= \int_0^1 \frac{1}{x^2+1}\,dx$ $= [\tan^{-1}x]_0^1 = \tan^{-1}1 - \tan^{-1}0 = \frac{\pi}{4}$	1 mark: correct solution Note: this is a **show** question… and $\int \frac{1}{x^2+1}dx \neq \tan x$, and, $\tan\frac{\pi}{4} \neq 1$
(d)(ii)	$I_n + I_{n-1} = \int_0^1 \left(\frac{x^{2n}}{x^2+1} + \frac{x^{2n-2}}{x^2+1}\right)dx$ $= \int_0^1 \left(\frac{x^{2n-2}(x^2+1)}{x^2+1}\right)dx$ $= \int_0^1 x^{2n-2}\,dx$ $= \left[\frac{1}{2n-1}x^{2n-1}\right]_0^1$ $= \frac{1}{2n-1}(1-0) \quad = \quad \frac{1}{2n-1}$	2 marks: correct solution 1 mark: substantially correct solution *Writes the sum as a single integral, or equivalent merit*
(d)(iii)	$\int_0^1 \frac{x^4}{x^2+1}\,dx = I_2$ $I_n + I_{n-1} = \frac{1}{2n-1} \quad \rightarrow \quad I_2 + I_1 = \frac{1}{2(2)-1} = \frac{1}{3}$ $I_1 + I_0 = \frac{1}{2(1)-1} = 1$ $subtracting\ \ldots \quad I_2 - I_0 = -\frac{2}{3}$ $I_2 = I_0 - \frac{2}{3}$ $I_2 = \frac{\pi}{4} - \frac{2}{3}$	2 marks: correct solution 1 mark: substantially correct solution *Attempts to apply the recursion relation from part (ii), or equivalent merit.*

2014 HSC examiners comments

(d)(i) Most candidates showed the correct working.

A common problem was:

- writing $\int_0^1 \frac{1}{x^2+1}\,dx = [\tan x]_0^1$ instead of $[\tan^{-1} x]_0^1$.

(d)(ii) The most common approach was writing $I_n + I_{n-1} = \int_0^1 \frac{x^{2n}}{x^2+1} + \frac{x^{2n-2}}{x^2+1}\,dx$, which after factorisation and cancellation reduced to $\int_0^1 x^{2n-2}\,dx = \frac{1}{2n-1}$.

Another method which was usually done well was $\int_0^1 \frac{x^{2n}}{x^2+1}\,dx = \int_0^1 \frac{x^{2n-2}(x^2+1)}{x^2+1}\,dx - \int_0^1 \frac{x^{2n-2}}{x^2+1}\,dx$. This resulted in the statement $I_n = \int_0^1 x^{2n-2}\,dx - I_{n-1}$ which eventually yields the correct answer of $I_n + I_{n-1} = \frac{1}{2n-1}$.

A common problem was:

- using the method of integration by parts, which led to a very convoluted proof (done poorly by all but a very few students).

(d)(iii) This part of the question was done successfully using part (ii), commencing with $I_1 + I_0 = \frac{1}{2\times1-1}$, hence $I_1 = 1 - \frac{\pi}{4}$ and then using $I_2 + I_1 = \frac{1}{2\times2-1}$ which eventually leads to the correct answer.

Some candidates used the method of long division where $\frac{x^4}{x^2+1} = x^2 - 1 + \frac{1}{x^2+1}$ which leads to a correct answer of $\frac{\pi}{4} - \frac{2}{3}$.

A common problem was:

- attempting to evaluate $\int_0^1 \frac{x^4}{x^2+1}\,dx$ as I_4, not I_2 as required.

Year 12	Mathematics Extension 2	Ass Task 4 2024 HSC
Question No. 16	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MEX 12-5 applies techniques of integration to structured and unstructured problems.
MEX 12-6 uses mechanics to model and solve practical problems.

A note on questions 15 & 16
All parts were taken from past HSC papers, with the intended purpose of "quality control", and to aid with your preparation for what is looked for by HSC markers.
The HSC examiners notes from those questions have been included here, so that you can see what they highlighted when marking, and compare with your own responses.
In particular, if you did not attain full marks for any part, you should read the examiners comments, as it was noted in the marking of this 2024 cohort that there were a lot of commonalities with errors or misjudgments between the past and the present.

Part / Outcome	Solutions	Marking Guidelines
(a) **12-5** **12-6**	$a = v^2 + v$ $\therefore \quad v\dfrac{dv}{dx} = v^2 + v$ $\therefore \quad \dfrac{dx}{dv} = \dfrac{1}{v+1}$ $x = \ln(v+1) + C$ $x = 0, v = 1 \quad \Rightarrow \quad C = -\ln 2$ $\therefore \quad x = \ln(v+1) - \ln 2$ alternate $v\dfrac{dv}{dx} = v^2 + v$ $\dfrac{v}{v^2+v}dv = dx$ $\displaystyle\int_1^v \frac{1}{v+1}dv = \int_0^x dx$ $[\ln\lvert v+1\rvert]_1^v = [x]_0^x$ $\ln\lvert v+1\rvert - \ln 2 = x$ $\therefore x = \ln\dfrac{\lvert v+1\rvert}{2}$	3 marks: correct solution 2 marks: substantially correct solution *Correctly separates variables, or equivalent merit* 1 mark: partially correct solution *Uses $a = v\frac{dv}{dx}$, or equivalent merit* *Note: x in terms of v* *Writing v in terms of x as final answer prevented full marks*

2020 X2 HSC examiners comments

Students should:

- use $v\frac{dv}{dx}$ to find an expression for x and evaluate the constant using the given conditions.

In better responses, students were able to:

- convert acceleration to $v\frac{dv}{dx}$
- manipulate and simplify $v\frac{dv}{x} = v^2 + v$ to gain $\frac{dx}{dv} = \frac{1}{v+1}$
- integrate and gain an expression in $\ln(v+1)$
- interpret the initial conditions to evaluate the constant.

Areas for students to improve include:

- using the most appropriate expression for acceleration
- simplifying an algebraic expression before integration
- calculating the constant of integration using the given information.

Part / Outcome	Solutions	Marking Guidelines
(b) **12-5** **12-6**	$\ddot{x} = v\dfrac{dv}{dx} = 2 - e^{-\frac{x}{2}}$ $\displaystyle\int_4^v v\,dv = \int_0^x \left(2 - e^{-\frac{x}{2}}\right)dx$ $\left[\dfrac{v^2}{2}\right]_4^v = \left[2x + 2e^{-\frac{x}{2}}\right]_0^x$ $\dfrac{v^2}{2} - \dfrac{16}{2} = \left(2x + 2e^{-\frac{x}{2}}\right) - \left(2(0) + 2e^{-\frac{0}{2}}\right)$ $\dfrac{v^2}{2} = 2x + 2e^{-\frac{x}{2}} - 2 + 8$ $v^2 = 4x + 4e^{-\frac{x}{2}} + 12$ alternate $\dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = \ddot{x} = 2 - e^{\frac{-x}{2}}$ $\dfrac{1}{2}v^2 = \displaystyle\int 2 - e^{-\frac{x}{2}}dx$ $= 2x - \dfrac{e^{\frac{-x}{2}}}{-\frac{1}{2}} + c$ $= 2x + 2e^{\frac{-x}{2}} + c$ When $x = 0, v = 4$ $\dfrac{1}{2}(16) = 0 + 2 + c$ $c = 6$ $\therefore \dfrac{1}{2}v^2 = 2x + 2e^{-\frac{x}{2}} + 6$ $v^2 = 4x + 4e^{\frac{-x}{2}} + 12$	3 marks: correct solution 2 marks: substantially correct solution *Obtains expression for v^2 possibly involving an undetermined constant* 1 mark: partially correct solution *Attempts to use $a = v\frac{dv}{dx}$, or $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$, or equivalent merit*

(c) Candidates generally knew that $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ or $v\frac{dv}{dx}$ and applied this correctly to attain the desired result. Some candidates approached the question by separating variables and were generally successful.

Common problems were:

- not finding the correct primitive of $e^{-\frac{x}{2}}$
- not determining the value of the constant of integration correctly
- finding $\dot{x} = v = 2x + 2e^{-\frac{x}{2}} + c$ and then squaring to get v^2
- finding $\frac{1}{2}v^2 = 2x + 2e^{-\frac{x}{2}} + 6$ but then writing $v^2 = x + e^{-\frac{x}{2}} + 3$.

(c)(i)

12-6

$$\ddot{x} = -4(x-3)$$

$\therefore n^2 = 4$, centre is $x = 3$

$n = 2$

Period $= \frac{2\pi}{n} = \frac{2\pi}{2} = \pi$

$$v^2 = n^2(a^2 - (x-c)^2)$$

When $v = 8, x = 0$

So, $64 = 4(a^2 - (0-3)^2)$

$16 = a^2 - 9$

$a^2 = 25$

$a = 5$

$\therefore$ the particle oscillates between

$x = 3 + 5$ and $x = 3 - 5$

$x = 8$ $\qquad$ $x = -2$

alternate

$v\frac{dv}{dx} = -4(x-3)$

$\int_8^v v\,dv = -4\int_0^x (x-3)\,dx$

$\left[\frac{v^2}{2}\right]_8^v = -4\left[\frac{x^2}{2} - 3x\right]_0^x$

$\frac{v^2}{2} - \frac{64}{2} = -4\left[\frac{x^2}{2} - 3x\right]$

$\frac{v^2}{2} = -2x^2 + 12x + 32$

$v^2 = -4x^2 + 24x + 64 = 0$ for extremeties

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = -2$ & 8

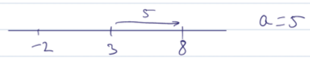
2 marks: correct solution

1 mark: substantially correct solution

States the amplitude of motion, or equivalent merit

(c)(ii)

12-6



$$x = a\cos(nt + \alpha) + c \qquad (a = 5, n = 2, c = 3)$$

$$x = 5\cos(2t + \alpha) + 3$$

$$5.5 = 5\cos\alpha + 3 \qquad (when\ t = 0, x = 5.5)$$

$$\therefore \alpha = \cos^{-1}\frac{2.5}{5}$$

$$= \frac{\pi}{3} = 1.04719\ldots \text{ radians}$$

$$0 = 5\cos(2t + 1.04719) + 3 \qquad (when\ x = 0)$$

$$\cos(2t + 1.04719) = \frac{-3}{5}$$

$$2t + 1.04719 = 2.214\ldots. \text{ radians}$$

$$2t = 1.167\ldots$$

$$t = 0.583$$

$\therefore$ The first value of t when $x = 0$ is $t = 0.58$ (2 decimal places)

2 marks: correct solution

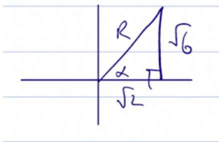
1 mark: substantially correct solution

Finds the displacement function, or equivalent merit

Interestingly… no examiners comments were provided for this question in 2021, however

A common problem in part (ii) was candidates choosing displacement to be a function in +sine… which leaves the velocity function (derivative) in terms of +cosine. The resulting acute angle gives x *and* $\dot{x}$ both positive. It required the second quadrant angle to be used in order give a negative $\dot{x}$. (Initial condition)

Choosing displacement to be a cosine function results in velocity function with opposite sign, creating negative velocity with the acute angle (as in these solutions). *(Does this make sense?)*

(d)(i)	$R\cos(3t-\alpha) = R\cos 3t\cos\alpha + R\sin 3t\sin\alpha$ $= \sqrt{2}\cos 3t + \sqrt{6}\sin 3t$ $R\cos\alpha = \sqrt{2} \quad \Rightarrow \quad \cos\alpha = \frac{\sqrt{2}}{R}$ $R\sin\alpha = \sqrt{6} \quad \Rightarrow \quad \sin\alpha = \frac{\sqrt{6}}{R}$  $R = 2\sqrt{2}, \qquad \alpha = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$ $\therefore x = 2\sqrt{2}\cos\left(3t - \frac{\pi}{3}\right)$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution *Finds R, or equivalent merit*
(d)(ii) **12-6**	At rest at extremities, ie when $x = \pm 2\sqrt{2}$	<u>1 mark</u>: correct answer**<u>s</u>** *Needed both correct values*
(d)(iii) **12-6**	$x = 2\sqrt{2}\cos\left(3t - \frac{\pi}{3}\right)$ $v = -6\sqrt{2}\sin\left(3t - \frac{\pi}{3}\right)$ $\therefore$ Max speed is $6\sqrt{2}$, and so $\frac{1}{2}$ max speed is $3\sqrt{2}$ $-6\sqrt{2}\sin\left(3t - \frac{\pi}{3}\right) = 3\sqrt{2}$ $\sin\left(3t - \frac{\pi}{3}\right) = -\frac{1}{2}$ $3t - \frac{\pi}{3} = -\frac{\pi}{6}$ $3t = \frac{\pi}{6}$ $t = \frac{\pi}{18}\ sec$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution *Finds half maximum speed, or equivalent merit* *Note: letting x=0 to find <u>when</u> the max speed happened is not enough for a mark. <u>What</u> the max speed was the important starting point*

<u>2019 HSC **(X1)** examiners comments</u>

(i)

Students should:

- identify when to use the auxiliary angle/transformation method to solve trigonometric equations.

In better responses, students were able to:

- find the values for R and α (in exact form) efficiently.

Areas for students to improve include:

- applying the auxilliary angle/transformation method skillfully
- knowing and using exact radian values.

(ii)

Students should:

- use the mark value of a question as a guide to the complexity of solution required.

In better responses, students were able to:

- realise that the particle would be at rest at the end points of its motion and that this equated to the amplitude of their expression from part (i)

Areas for students to improve include:

- understanding the features of Simple Harmonic Motion
- reading the question carefully in order to answer the question that is asked.

(iii)

Students should:

- understand the interrelatedness of the parts of a question
- use their result from part (i) to obtain an expression for velocity and solve a trigonometric equation to find a time.

In better responses, students were able to:

- realise that maximum velocity would be the amplitude of their expression for velocity
- understand that speed would be a positive value
- deduce that a 4^{th} quadrant result to their trigonometric equation would still yield a positive value for time.

Areas for students to improve include:

- understanding the difference between speed and velocity
- developing their ability to solve trigonometric equations and working with exact trigonometric values.

A final thought on the marking…

Particularly for 2 mark questions:
Quite often, candidates did a large amount of work in some questions, and received no marks… this was quite often the case for a two mark question.
Reading through HSC examiners comments and marking schemes over the years points to the fact that the first mark is awarded quite deep into the solution.
As a general 'rule of thumb', in Extension 2, to get that first mark you generally need to "be on the right track" with the deeper understanding that is required for this course. The second mark is essentially awarded for the execution.

Hand writing was particularly bad (careless), often creating ambiguous or illegible symbols – preventing the validity of solutions to be conveyed effectively.
If you want the marks, you need to *earn* them. This includes clarity with handwriting when so many different symbols are involved, and they have significantly different meanings.